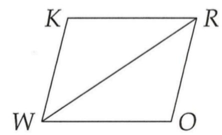
8.3.1

(a) Opposite sides in a parallelogram are equal. Therefore, $IU = TY = 6$ and $TI = YU = 8$. The perimeter of the parallelogram is then $6 + 8 + 6 + 8 = \boxed{28}$.

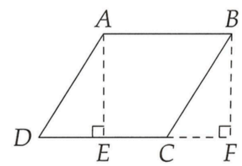
(b) $\boxed{\text{No}}$. The area of a parallelogram is not determined by the side lengths of the parallelogram. Below are two parallelograms with the same perimeter, but with very different areas.

8.3.2 Since $\overline{WX} \parallel \overline{YZ}$, we have $\angle W + \angle Z = 180^\circ$. Similarly, $\angle Y + \angle Z = 180^\circ$ because $\overline{XY} \parallel \overline{WZ}$. Therefore, $\angle W = \angle Y$.

8.3.3 Since $\overline{WO} \parallel \overline{RK}$, and $\overline{WR}$ is a transversal cutting the two parallel segments, we have $\angle OWR = \angle KRW$. We also have $WO = RK$ and $WR = WR$. Hence, triangles OWR and KRW are congruent by SAS. Therefore, $\angle ORW = \angle KWR$, and this implies $\overline{WK} \parallel \overline{RO}$. The opposite sides of $WORK$ are then parallel, so it must be a parallelogram.



8.3.4 Let $ABCD$ be a parallelogram. Let E and F be the feet of the perpendiculars from A and B to $\overleftrightarrow{CD}$, respectively. Then triangles AED and BFC are right triangles, and $ABFE$ is a rectangle, so $AE = BF$. Since $ABCD$ is a parallelogram, we have $AD = BC$, so by HL congruence, triangles AED and BFC are congruent. Therefore,



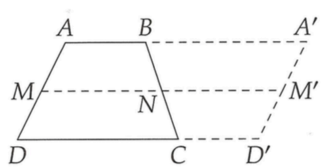
$$[ABCD] = [ABCE] + [AED] = [ABCE] + [BFC] = [ABFE].$$

In other words, the areas of parallelogram $ABCD$ and rectangle $ABFE$ are equal. The area of rectangle $ABFE$ is the base AB times the height BF. Therefore, the area of the parallelogram $ABCD$ is also the base AB times the height BF.

8.3.5 Point E is on line $\overleftrightarrow{EU}$ and point N is on line $\overleftrightarrow{NT}$, and the distance between lines $\overleftrightarrow{EU}$ and $\overleftrightarrow{NT}$ is given as 5. This means that the closest any point on one line can be to the other is 5 units. Therefore, the length of $\overline{EN}$ must be at least 5, and in particular, it cannot be 4.

8.3.6

(a) We have $AA' = AB + BA' = AB + CD$ and $DD' = DC + CD' = AB + CD$, so $AA' = DD'$. Furthermore, $\overline{AA'} \parallel \overline{DD'}$. As we saw in Exercise 8.3.3, $AA'D'D$ is a parallelogram because $AA' = DD'$ and $\overline{AA'} \parallel \overline{DD'}$.



(b) Consider trapezoids $ABCD$ and $D'CBA'$. By construction, $D'C = AB$ and $BA' = CD$. Since $AA'D'D$ is a parallelogram, $A'D' = DA$. Therefore, corresponding sides of the two trapezoids are equal. Furthermore, our parallel segments give us $\angle DAB = \angle A'D'C$, $\angle ABC = \angle D'CB$, $\angle BCD = \angle CBA'$, and $\angle CDA = \angle BA'D'$. Thus, corresponding angles of the two trapezoids are also equal, so they are congruent.

Therefore, $[ABCD] = [D'CBA']$. But $[ABCD] + [D'CBA'] = [AA'D'D]$, so $[AA'D'D] = 2[ABCD]$. Finally, the area of the parallelogram is $[AA'D'D] = AA' \cdot h = (AB + CD) \cdot h$, so

$$[ABCD] = \frac{1}{2}[AA'D'D] = \frac{1}{2}(AB + CD) \cdot h.$$

(c) We have $AM = AD/2 = A'D'/2 = A'M'$, and $\overline{AM} \parallel \overline{A'M'}$. Hence, $AA'M'M$ is a parallelogram, so $MM' = AA' = AB + CD$. However, $\overline{MN}$ and $\overline{M'N}$ are corresponding parts of congruent trapezoids $ABCD$ and $D'CBA'$, so $MN = M'N$. Since $MM' = MN + NM'$, we have $MN = MM'/2 = (AB + CD)/2$.

8.4.1 As shown earlier, the area of a rhombus is half the product of its diagonals. Hence, the area of $PQRS$ is $(6)(12)/2 = \boxed{36}$.

The diagonals bisect each other and are perpendicular, thus cutting the rhombus into four right triangles with legs $6/2 = 3$ and $12/2 = 6$. The hypotenuses of these triangles are the sides of the rhombus, and from the Pythagorean Theorem each has length $\sqrt{3^2 + 6^2} = 3\sqrt{5}$, so the perimeter of the rhombus is $4(3\sqrt{5}) = \boxed{12\sqrt{5}}$.

8.4.2

(a) The diagonals bisect each other, cutting the rhombus into four right triangles of hypotenuse $WX = 50$ and one leg $WY/2 = 48$. Then the other leg is $\sqrt{50^2 - 48^2} = \sqrt{196} = 14$, so diagonal $XZ = 2 \cdot 14 = \boxed{28}$.

(b) The area of the rhombus is $(28)(96)/2 = \boxed{1344}$.

(c) Since $WXYZ$ is a rhombus, it is a parallelogram. Therefore, its area equals the product of the length of one side and the distance from that side to the opposite side. Since the area of $WXYZ$ is 1344 and $WX = 50$, the distance (height) between $\overline{WX}$ and $\overline{YZ}$ is $1344/50 = \boxed{672/25}$.

8.4.3 Let E be the intersection of the diagonals. Then

$$\begin{aligned}
[ABCD] &= [AEB] + [BEC] + [CED] + [DEA] \\
&= \frac{1}{2}AE \cdot EB + \frac{1}{2}BE \cdot EC + \frac{1}{2}CE \cdot ED + \frac{1}{2}DE \cdot EA \\
&= \frac{1}{2}(AE \cdot EB + BE \cdot EC + CE \cdot ED + DE \cdot EA) \\
&= \frac{1}{2}(AE + EC)(BE + ED) \\
&= \frac{1}{2}AC \cdot BD.
\end{aligned}$$

8.4.4

(a) Triangles TUW and VUW are congruent by SSS. Therefore, $\angle TUW = \angle VUW$. Since the two angles combined make 60°, each angle is 30°.

(b) Diagonal $\overline{TV}$ cuts the rhombus into two equilateral triangles of side 10. Hence, $[TUVW] = 2[TUV] = 2(10^2\sqrt{3})/4 = \boxed{50\sqrt{3}}$.

8.5.1

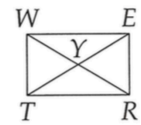
(a) Opposite sides in a rectangle are equal. Therefore, the perimeter is $8 + 12 + 8 + 12 = \boxed{40}$.

(b) The segments $\overline{PS}$, $\overline{PO}$, and $\overline{OS}$ make a right triangle with $\overline{PS}$ as hypotenuse. Therefore,

$$PS = \sqrt{PO^2 + OS^2} = \sqrt{8^2 + 12^2} = \sqrt{208} = \boxed{4\sqrt{13}}.$$

(c) The area of rectangle $POST$ is simply the length times the width, which is $8 \cdot 12 = \boxed{96}$.

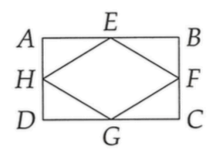
8.5.2 Let the width of the rectangle be x. Then the length of the rectangle is $2x - 1$, and the perimeter is $2[x + (2x - 1)]$. Setting this equal to 36 gives $x = 19/3$. The area of the rectangle is then $x(2x - 1) = (19/3)(35/3) = \boxed{665/9}$.



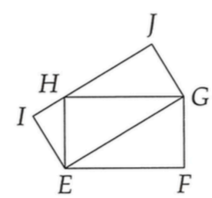
8.5.3 Triangle EYR is isosceles with $YE = YR$, so $\angle ERY = \angle YER$. Angle $\angle WYE = x$ is an exterior angle of triangle EYR, so $\angle ERY + \angle YER = \angle WYE = x$, from which we have $\angle ERY = \boxed{x/2}$. Then $\angle YRT = \angle ERT - \angle ERY = 90^\circ - \angle ERY = \boxed{90^\circ - x/2}$.

8.5.4 The other diagonal of the rectangle is a radius of the circle, which is 9 cm. The two diagonals in a rectangle are equal, so $RQ = \boxed{9 \text{ cm}}$.

8.5.5 The frame has length $36 + 2 + 2 = 40$ and width $24 + 2 + 2 = 28$, so the area of the frame is $40 \cdot 28 - 36 \cdot 24 = 256$ square inches. The cost of the frame is then $\$1.50 \cdot 256 = \boxed{\$384}$.



8.5.6 Let $ABCD$ be a rectangle, and let E, F, G, and H be midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. In triangles AEH and BEF, we have $AH = AD/2 = BC/2 = BF$, $AE = AB/2 = BE$, and $\angle A = 90^\circ = \angle B$. Hence, the two triangles are congruent by SAS. Therefore, $HE = EF$. Similarly, $EF = FG$, $FG = GH$, and $GH = HE$. Thus, all four sides of $EFGH$ are equal, so $EFGH$ is a $\boxed{\text{rhombus}}$.



8.5.7 Triangle HEG and rectangle $IEGJ$ share the same altitude from H to $\overline{EG}$, so $[IEGJ] = 2[HEG]$. On the other hand, triangle HEG is half of rectangle $EFGH$, so $[EFGH] = 2[HEG]$. Therefore, $[IEGJ] = [EFGH] = \boxed{48}$.

8.6.1 $EF = \sqrt{80} = \boxed{4\sqrt{5}}$, and $EG = EF\sqrt{2} = \boxed{4\sqrt{10}}$.

8.6.2 Since $BM/BA = BO/BD = 1/2$ and $\angle MBO = \angle ABD$, triangles BMO and BAD are similar with ratio 1/2 by SAS Similarity, so $AD = 2MO = 8$. Therefore, the area of square $ABCD$ is $8^2 = \boxed{64}$.

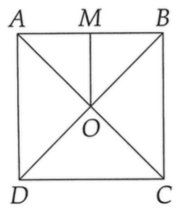


Figure 8.1: Diagram for Problem 8.6.2

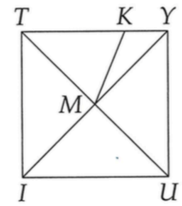


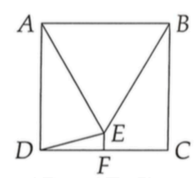
Figure 8.2: Diagram for Problem 8.6.3

8.6.3 First, $\angle MTK = \angle UTY = \boxed{45^\circ}$. Since $TK = TM$, we have $\angle MKT = \angle TMK$. From $\triangle MTK$, we have $\angle MKT + \angle TMK = 180^\circ - \angle MTK = 135^\circ$, so $\angle TMK = 135^\circ/2 = \boxed{67.5^\circ}$.

8.6.4 Since every rectangle is a parallelogram, the diagonals of a rectangle bisect each other. Therefore, if the diagonals of a rectangle are perpendicular, they divide the rectangle into four triangles that are congruent by SAS Congruence. The hypotenuses of these triangles, which are the sides of the rectangle, are therefore equal in length. Thus, the sides of the rectangle are all equal in length, so the rectangle is a square.

8.6.5 Since $\overline{AC}$ is a diagonal of square $ABCD$, $AC = AB\sqrt{2}$. Therefore, $[ACFG]/[ABCD] = AC^2/AB^2 = (AC/AB)^2 = \boxed{2}$.

8.6.6



(a) Since triangle ABE is equilateral, $AE = AB = \boxed{4}$.

(b) $ABCD$ is a square of side length 4, so its area is $4^2 = \boxed{16}$.

(c) ABE is an equilateral triangle of side length 4, so its area is $4^2\sqrt{3}/4 = \boxed{4\sqrt{3}}$.

(d) $\angle DAE = \angle DAB - \angle EAB = 90^\circ - 60^\circ = \boxed{30^\circ}$. Triangle DAE is isosceles with $DA = AB = AE$. Since $\angle DAE = 30^\circ$, $\angle DEA = (180^\circ - \angle DAE)/2 = (180^\circ - 30^\circ)/2 = \boxed{75^\circ}$.

(e) The area inside square $ABCD$ but outside triangle ABE is simply $\boxed{16 - 4\sqrt{3}}$.

(f) Let F be the foot of perpendicular from E to $\overline{CD}$, so $FC = 2$. (Make sure you see why F must be the midpoint of $\overline{CD}$.) The altitude from E to $\overline{AB}$ in triangle AEB is $(\sqrt{3}/2)AE = 2\sqrt{3}$, so $EF = 4 - 2\sqrt{3}$. Then by the Pythagorean Theorem applied to right triangle EFC, we have

$$EC^2 = EF^2 + FC^2 = (4 - 2\sqrt{3})^2 + 2^2 = 32 - 16\sqrt{3}.$$

We then observe that $(2\sqrt{6} - 2\sqrt{2})^2 = 32 - 16\sqrt{3}$, so $EC = \boxed{2\sqrt{6} - 2\sqrt{2}}$.